chapter 4 Integration

## Section 4.6 Average value of a function

PROJECT MATHS Text & Tests 7



Average ("y") Value in a region

$A = LB$

$A = xy$

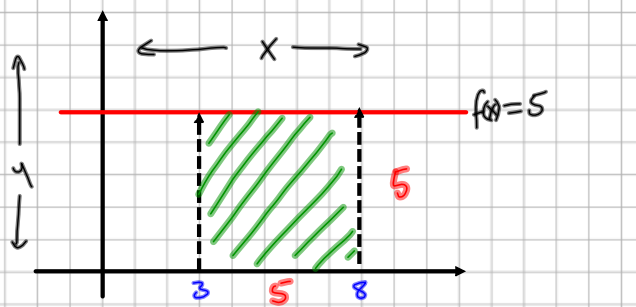
$y = \frac{A}{x}$

Average $y$ = $\frac{\text{Area}}{\text{width}}$

Area $= \int_a^b f(x)\,dx$

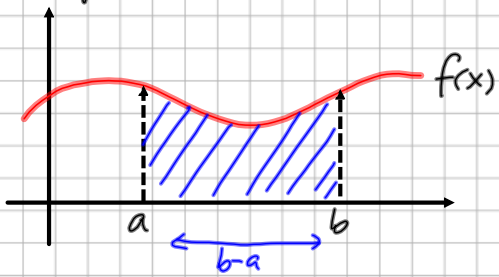
Average value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

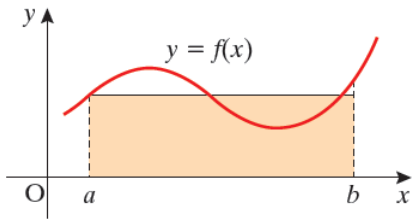
$\frac{1}{\text{width}} \cdot \text{Area}$




$A = \int_3^8 5\,dx = 25$

Average Value = $A/(x\text{-width}) = \frac{25}{5} = 5$






The average value of a function $f(x)$ over the interval $[a, b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

learn formula

## Example 2

(distance in metres)

A body starts from rest and moves in a straight line.
After $t$ seconds its velocity ($v$) is given by $v = 2t - 4, t \geqslant 0$.

(i) By completing the table on the right, find the average velocity over the first 3 seconds.

| $t$ = | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $v$ = | −4 | −2 | 0 | 2 |

(ii) Use integration to test the accuracy of your answer.

(i)

$t=0 \Rightarrow v = 2(0) - 4 = -4$ m/s
$t=1 \Rightarrow v = 2(1) - 4 = -2$ m/s
$t=2 \Rightarrow v = 2(2) - 4 = 0$ m/s
$t=3 \Rightarrow v = 2(3) - 4 = 2$ m/s

Average $= \frac{-4-2+0+2}{4}$

Average $= -1$ m/s

(ii)

Average Value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

$a=0, b=3$
$b-a = 3-0=3$

Average Value $= \frac{1}{3}\int_0^3 (2t-4)\,dt$

$= \frac{1}{3}\left[\frac{2t^2}{2} - 4t\right]_0^3 = \frac{1}{3}\left[\left(\frac{3^2}{3} - 4(3)\right) - (0)\right]$

$= \frac{1}{3}[9-12] = -1$ m/s

**4.** Find the average value of the function $f(x) = x^2 + 4$ for $-2 \leq x \leq 3$.

$$\text{Average Value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$a = -2$

$b = 3$

$b - a = 3 - -2 = 5$

$$\text{Average Value} = \frac{1}{5}\int_{-2}^{3} (x^2+4)\,dx$$

$$= \frac{1}{5}\left[\frac{x^3}{3} + 4x\right]_{-2}^{3}$$

$$= \frac{1}{5}\left[\left(\frac{3^3}{3} + 4(3)\right) - \left(\frac{(-2)^3}{3} + 4(-2)\right)\right]$$

$$= \frac{19}{3}$$